Engineering Notes

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Symmetric Flow Characteristics of Thin Rectangular Wings

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Nomenclature

A = aspect ratio
B = $\sqrt{M^2-1}$
c = wing chord
C_N = $\dfrac{\text{normal force}}{qS}$ = normal force coefficient
K_p = $\dfrac{\partial(C_{N,p})}{\partial(\sin\alpha\cos\alpha)}$
$K_{v,\mathrm{le}}$ = $\dfrac{\partial(2\text{ leading-edge suction force from one side}/qS)}{\partial\sin^2\alpha}$
$K_{v,\mathrm{se}}$ = $\dfrac{\partial(2\text{ tip suction force from one side edge}/qS)}{\partial\sin^2\alpha}$
M = Mach number
q = freestream dynamic pressure
S = wing area
α = angle of attack, deg
β = $\sqrt{1-M^2}$

Subscripts

p = potential or attached flow
v, le = vortex effect at the leading edge
v, se = vortex effect at the side edge

Introduction

IN a previous Note[1] a set of semiempirical analytic formulas was reported which can serve as a solution of the extended[2] Multhopp method for the symmetric aerodynamic characteristics of sharp-edged rectangular wings. The two solutions give practically the same result for the lift, lift dependent drag, and pitching moment coefficients for $M\leq 1$. This is a consequence of the numerically[2] obtained coefficients K_p, $K_{v,\mathrm{le}}$ and $K_{v,\mathrm{se}}$, in Polhamus'[3] and Lamar's[2] nomenclature, being represented accurately by analytic expressions.[1] The formulas correlate with K_p and $K_{v,\mathrm{le}}$ mostly within printing accuracy and correlate with $K_{v,\mathrm{se}}$ with hardly less accuracy, as can be seen from Fig. 1, repeated here from Ref. 1. The diverging discrepancy in $K_{v,\mathrm{se}}$ for increasing reduced aspect ratios when $\beta A>1.5$ is more of theoretical than practical interest, because the side force contribution to the total coefficient decreases for increasing aspect ratio. For $\beta A<1.5$ a wavy discrepancy between the two $K_{v,\mathrm{se}}$ representations can be noticed and it is impossible to judge which one is the most realistic.

It is interesting to see how the Mach number dependence in the analytic expressions,[1] prepared for $M\leq 1$, stand compared with panel method results other than Lamar's.[2] Recent theoretical results for $A=2$ by Lan and Mehrotra,[4] which also compare the results of Woodward[5] and Lamar,[2] are now available for such a comparison. For the sake of completeness, panel method results[4,5] and results of the exact linear supersonic theory by Harmon[6] and Margolis[7] are also included in order to illustrate the performance of linearized theory in this respect.

Analytic Formulas

The analytic expressions[1] for the coefficients K_p, $K_{v,\mathrm{le}}$, and $K_{v,\mathrm{se}}$ to be compared at $M\leq 1$ with the panel method results are:

$$\beta K_p = 2\pi\beta A/[2+\sqrt{(4/3)(\beta A)^2+4}] \tag{1}$$

$$\beta K_{v,\mathrm{le}} = \pi\beta A/[2+\sqrt{(1/4)(\beta A)^2+4}] \tag{2}$$

$$K_{v,\mathrm{se}} = 2\pi/(2+\beta A) \tag{3}$$

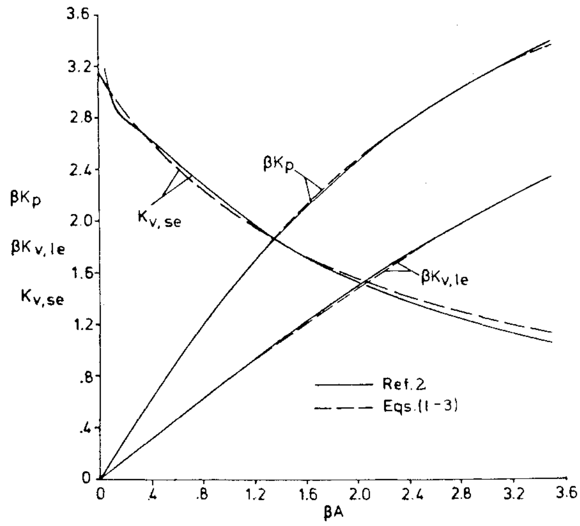

Fig. 1 Variation of βK_p, $\beta K_{v,\mathrm{le}}$, and $K_{v,\mathrm{se}}$ with aspect ratio and Mach number for rectangular wings according to Lamar[1] compared with the semiempirical expressions, Eqs. (4-7).

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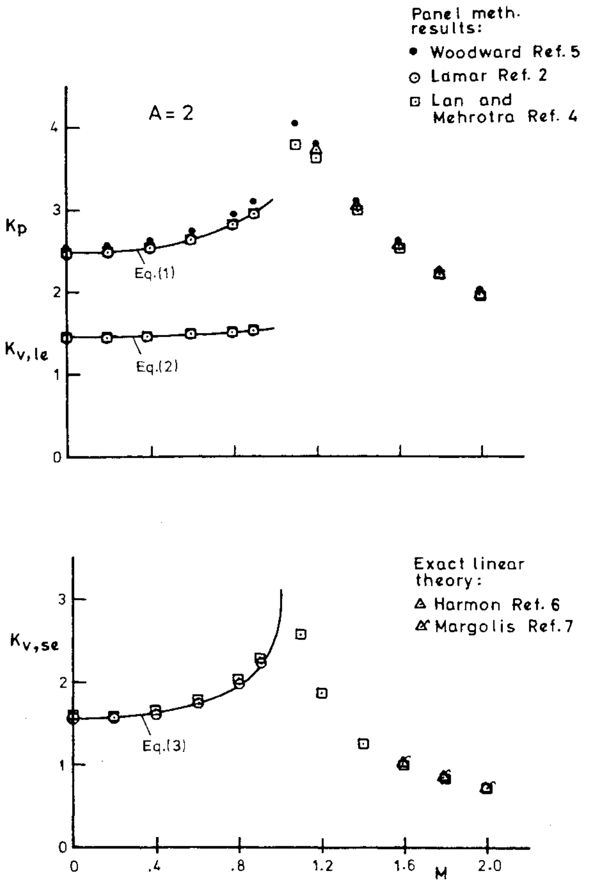

Fig. 2 The coefficients K_p, $K_{v,le}$, and $K_{v,se}$ as functions of the Mach number obtained by different panel methods and by exact linear supersonic theory for a rectangular wing with aspect ratio $A = 2$. The subsonic results are compared with the semiempirical expressions, Eqs. (1-3).

Equations (1-3) are derived for sharp-edged rectangular wings and it is believed that the constants, 4/3 and 1/4, in Eqs. (1) and (2) could be functions of the leading-edge radius. Such functions are, however, not known at present.

From the exact linear supersonic theory, coefficients corresponding to Eqs. (1-3) have been obtained by Harmon[6] and Margolis.[7] The results written in the nomenclature[2,3] above are thus for $M > 1$:

$$K_p = (4/B)(1 - 1/2BA) \tag{4}$$

$$K_{v,le} = 0 \tag{5}$$

$$K_{v,se} = 8/\pi BA \tag{6}$$

where $B = \sqrt{M^2 - 1}$ and the first term in Eq. (4) is Ackeret's result for a two-dimensional wing in supersonic flow.

The set of Eqs. (1-6) summarizes in a concise analytic form the knowledge that at present is obtained from linearized theory as to the total, stationary, symmetric forces on thin rectangular wings. The result is not restricted to small angles of attack but, thanks to the suction analogy,[3] can be used as long as the flow reattaches on the wing after having separated from the edges of the wing. Strong shocks and vortex breakdown, however, do set limits for the applicability of Eqs. (1-6).

Comparsion of Results and Discussion

Equations (1-3), applied to a wing with aspect ratio $A = 2$, are plotted in Fig. 2 for a comparison with the panel method results[2,4,5] in the transonic region $M \leq 1$, in the first place. The improved Woodward[5] panel method by Lan and Mehrotra[4] very nearly reproduces Lamar's[2] results for the coefficients K_p and $K_{v,le}$, as can be seen in Fig. 2. From the tabulated results[4] it can be determined that the discrepancy is less than 0.004 in K_p and slightly less than that in $K_{v,le}$, the Lan and Mehrotra K_p being systematically the highest, and their $K_{v,le}$ systematically the lowest. The early panel method result for K_p by Woodward[5] is systematically higher than the above results (from about 0.1 at $M = 0$ to about 0.13 at $M = 0.9$), as can be seen in Fig. 2. Equations (1) and (2) are thus quantitatively very well supported by the panel method results and the small discrepancy at intermediate βA in Fig. 1 is just on the edge of being discernable in Fig. 2 for Mach numbers around $M = 0.6$.

$K_{v,se}$ obtained by Lan and Mehrotra[4] is systematically from 0.03 ($M = 0$) to 0.04 ($M = 0.9$) larger than Lamar's result and supports Eq. (3) at low Mach numbers. With increasing Mach numbers, Eq. (3) slightly but increasingly underestimates Lamar's result, which, in its turn, behaves in the same manner towards $K_{v,se}$ obtained by Lan and Mehrotra. At present it is not possible to decide which of the three results is closest at increasing Mach number to a solution of the full potential flow equation.

Conclusions

The semiempirical analytic expressions, Eqs. (1-3), contain in themselves Mach number influences that are in very good agreement with the investigated panel method results for $M \leq 1$, the intended regime of application of Eqs. (1-3). The differences between Eqs. (1-3) and the theoretical results are small and in fact only of minor significance when shortcut evaluations are wanted. However, a comparison with relevant transonic experimental results, if such results existed, would show an appreciable discrepancy in the transonic regime as a result of the neglect of genuine transonic terms in the linearized theory.

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